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STURM THEOREMS FOR SECOND ORDER LINEAR NONHOMOGENOUS DIFFERENTIAL EQUATIONS AND LOCALIZATION OF ZEROS OF THE SOLUTION

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Abstract. It is shown that Sturm theorems, formulated in the 1830's ([1], [2], [3] and [4]) and valid for second order linear homogeneous differential equation $L(y) \equiv y'' + a(x)y' + b(x)y = 0$, could as well be formulated for the class of nonhomogeneous linear differential equations $L(y) = f(x)$. Criteria for the existence of oscillatory solutions of nonhomogeneous equations, as well as more exact locations of the zeros are given.

1. Introduction and preliminaries

Sturm theorems are qualitative statements basically determining the relative positions of zeros and extremes of second order oscillations. Yet, they do not provide more exact locations of zeros. 180 years had passed since the initial formulation of the theorems without a significant progress in the literature, so we concluded that there was a necessity for more exact locations of zeros as well as other details of the behavior of the solutions $y_{1,2}$ and their derivatives $y'_{1,2}$. Sturm (who wrote the second order equation in its specific form $\big(p(x)y'\big)' + q(x)y = 0$) failed to obtain better results due to the following reasons:

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